

Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2011-2012
Backpaper Examination : Statistics (I)

Date: 31-12-2011

Maximum Score 80

Duration: 3 Hours

1. To establish a standard for parachute design, a researcher recorded the following fill times, in seconds, for 27 standard parachutes, obtained under controlled test conditions.

.59 .38 .47 .43 .44 .37 .43 .37 .27 .54 .39 .89 .48 .52
.51 .49 .38 .38 .23 .44 .40 .36 .33 .82 .51 .44 .37

- (a) Make a stem and leaf plot of these data.
- (b) Find the sample mean \bar{X} .
- (c) Give formula for sample standard deviation s . Do not compute.
- (d) Find the sample median M .
- (e) Find 100 p -th percentiles for $p = 0.2$ and 0.8 .
- (f) Find the first and third quartiles.
- (g) Draw the box plot and identify the outliers.
- (h) For trimming fraction 0.05 obtain the trimmed mean \bar{X}_T .
- (i) Explain how to obtain the trimmed standard deviation s_T . Do not compute.
- (j) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which the data arrived?

[4 + 2 + 2 + 2 + 4 + 4 + 5 + 3 + 3 + 4 = 33]

2. The independent random variables X_1, X_2, \dots, X_n have common distribution specified by

$$P(X \leq x | \alpha, \beta) = \begin{cases} 0 & \text{if } x < 0 \\ \left(\frac{x}{\beta}\right)^\alpha & \text{if } 0 \leq x \leq \beta \\ 1 & \text{if } x > \beta \end{cases}$$

where α, β are positive. It was found that the length of cuckoos' eggs found in hedge sparrow nests could be modelled with this distribution. Obtain *method of moments estimators* as well as *maximum likelihood estimators* for α, β .

[20]

[PTO]

3. Suppose you can draw a random sample from $U \sim \text{uniform}[0, 1]$. Explain how you would draw observations on a random variable W that has $\text{Beta}(m, n)$ distribution, where m and n are positive integers.

[10]

4. Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ be a random sample from the distribution with following bivariate density function

$$f_{XY}(x, y) = \frac{1}{2cd} I_{(-d, d)}(x) I_{(x^2, x^2+c)}(y)$$

where c, d are positive numbers.

Let $V = \frac{1}{n} \sum_{i=1}^n X_i$ and $W = \frac{1}{n} \sum_{i=1}^n Y_i$. Find ρ_{VW} , the correlation coefficient between V and W . What happens as $c \rightarrow 0$?

[10 + 2]

5. An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ square fluid ounces. If the variance of fill volume exceeds 0.01 square fluid ounces, an unacceptable proportion of bottles will be under or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with under and overfilled bottles? Use $\alpha = 0.05$. Report the p -value. Obtain 90% *Confidence Upper Bound (CUB)* for σ^2 .

[2 + 6 + 2 + 4 = 14]